

Second Edition

Introductory **ALGEBRA**

with **P****O****W****E****R** Learning

Sherri Messersmith
Nathalie M. Vega-Rhodes
Robert S. Feldman



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Education

Introductory ALGEBRA

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INTRODUCTORY ALGEBRA WITH P.O.W.E.R. LEARNING, SECOND EDITION

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About the Authors

Sherri Messersmith

Professor of Mathematics, College of DuPage



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Sherri Messersmith began teaching at the College of DuPage in Glen Ellyn, Illinois, in 1994 and has over 25 years of experience teaching many different courses from developmental mathematics through calculus. She earned a Bachelor of Science degree in the Teaching of Mathematics at the University of Illinois at Urbana-Champaign and taught high school for two years. Sherri returned to UIUC and earned a Master of Science degree in Applied Mathematics and stayed on at the university to teach and coordinate large sections of undergraduate math courses as well as teach in the Summer Bridge program for at-risk students. She is the author of many McGraw-Hill Education texts.

Sherri and her husband, Phil, are empty-nesters who recently relocated to the East Bay when her husband took a position at the University of California—Berkeley as a professor of Materials Science and Bioengineering. Sherri loves to cook, hang out with her dogs, study French, and travel; the manuscripts for this and her other books have accompanied her on her travels all over the globe.

Nathalie M. Vega-Rhodes

Professor of Mathematics, Lone Star College—Kingwood



©Nathalie M. Vega-Rhodes

Nathalie Vega-Rhodes' career in higher education began seventeen years ago and has encompassed a number of student-focused positions. For nearly a decade, she has taught mathematics ranging from developmental courses to calculus, as well as student success courses. She holds a Bachelor of Arts in Mathematics from the University of Houston and a Master of Science in Mathematics from the University of Houston—Clear Lake. In addition to teaching, as Mathematics Technology Coordinator at Lone Star College—Kingwood, she assists math faculty with technology-related pedagogical and implementation strategies. Her earliest work in higher education focused on academic support, first as a tutor and supplemental instruction (SI) leader, and then as a coordinator for a math tutoring and SI program. In her free time, Nathalie enjoys scuba diving and traveling with her husband, hanging out with her dog, and reading.

Robert S. Feldman

Deputy Chancellor and Professor of Psychological and Brain Sciences, University of Massachusetts—Amherst



©Robert S. Feldman

Bob Feldman still remembers those moments of being overwhelmed when he started college at Wesleyan University. “I wondered whether I was up to the challenges that faced me,” he recalls, “and although I never would have admitted it then, I really had no idea what it took to be successful at college.”

That experience, along with his encounters with many students during his own teaching career, led to a life-long interest in helping students navigate the critical transition that they face at the start of their own college careers. Bob, who went on to graduate with High Honors from Wesleyan and receive a Doctorate in Psychology from the University of Wisconsin—Madison, teaches at the University of Massachusetts—Amherst, where he is the Deputy Chancellor and Professor of Psychological and Brain Sciences. He is founding director of the first-year experience course for incoming students at UMass and is Senior Fellow of the Center for Student Success Research.

Bob is a Fellow of the American Psychological Association, the American Association for the Advancement of Science, and the Association for Psychological Science. He has written and edited more than 250 scientific articles, book chapters, and books, including *P.O.W.E.R. Learning: Strategies for Success in College and Life*, 7e; *Understanding Psychology*, 14e; and *The First Year of College: Research, Theory, and Practice on Improving the Student Experience and Increasing Retention*. He is past president of the FABBS Foundation, an umbrella group of societies promoting the behavioral and brain sciences, and he is on the Board of Directors of New England Public Radio. Bob loves travel, music, and cooking. He and his wife live near the Holyoke mountain range in western Massachusetts.

The Story of the P.O.W.E.R. Textbooks

In the classroom, I viewed the textbook as merely a *guide* and did many other things on my own to better meet the needs of my students. For example, I taught in bite-sized pieces because developmental students in particular learn better when material is presented in more manageable chunks. As students' basic skills deteriorated, I created several Basic Skills Worksheets to help improve those skills. The students did them in class, and it only took up two or three minutes of precious class time. When McGraw-Hill Education saw some of the materials that I had made for my own classroom, they asked if I would write a textbook. So in 2004 I began writing my first book, *Beginning & Intermediate Algebra*, now in its 4th edition. The material for the book was written to align with how students learn best, and these strategies and activities were included in both the book and its accompanying supplements. Twenty-five years of teaching and writing that first book revealed that *developmental students don't want to fail; they just don't know how to succeed*.

Over the years, the books have evolved to include everything possible to give students *and* instructors the tools they need for success. Explicit student success skills were not included as part of the first several books. But it became more and more clear over time that students needed direct instruction in skills like how to *effectively* do homework, how to read a math textbook, and how to manage their time. I had been addressing all of this in my own classroom, and after meeting Bob Feldman, I began doing it more formally by using his research-based P.O.W.E.R. Learning framework. I asked myself, "Why not incorporate P.O.W.E.R. into the textbooks to both teach study skills and organize the material according to the way research says students learn best?" Happily, Bob agreed to come on as a coauthor to give us the math textbooks with P.O.W.E.R. Learning that we have today.

Even with this major evolution in our approach, *we made sure not to compromise the math whatsoever!* The math content and its level of rigor remains the same as it was in the books written before we added the P.O.W.E.R. framework—so the fact that our books contain student success skills does *not* mean they are lacking in rigor. These books are light-years ahead of the earlier ones because we have now addressed a huge weakness of many college students: *knowing how to learn*.

With these 2nd edition Math with P.O.W.E.R. Learning books, we have evolved even more. The power (no pun intended) of the Internet is that it allows us to enhance the learning and teaching experience for students and instructors. We can do things that we couldn't dream of even 10 years ago! That's where Nathalie Vega-Rhodes comes in. A long-time P.O.W.E.R. math textbook user and outstanding instructor, Nathalie has a lot of experience using digital tools. We share the same philosophy of teaching, love for students, and belief in addressing *all* of our students' needs in the classroom, so why not bring her on board as a coauthor with digital expertise? So that's what we did. An immediate benefit was the creation of the Integrated Video & Study Guides, or IVSGs, new to this edition. It was a perfect collaboration; I wrote the IVSGs, and Nathalie made the new online videos.

Our team is complete, but this isn't the end; our story is not finished. The books continue to evolve and improve because of engaged faculty like you. We are so happy that you have chosen our textbook and want to continue the conversation. We would love to hear from you. Tell us your stories, and share your suggestions. Be a part of our story and part of the evolution of the P.O.W.E.R. math textbooks.

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Why should you use *Introductory Algebra with P.O.W.E.R. Learning*?

- **Solid, time-tested math content** with the amount of rigor needed to succeed in college-level courses
- **Written with friendly, conversational, non-intimidating language**, making it easier to read than most books while using all of the necessary mathematical language our students need
- **Written in bite-sized pieces** to make it easier to learn "complicated" material (For example, the rules of exponents are presented in their own chapter, Chapter 5.)
- **Engaging applications** written with students and their interests in mind
- **Rewritten, easy-to-use, research-based student success materials** in every chapter *in the book* that instructors can use at their discretion

- **Research-based pedagogical features throughout the books**
 - **24-hr problems:** Marked in each exercise set, these are indicated by an icon. Even if students do not complete their *entire* homework assignment immediately, they are encouraged to do the 24-hr problems within 24 hours of leaving class so that they better retain what they have learned.
 - **“You Try” exercises:** After almost every example in the book, students can do a You Try problem to work out a problem that is similar to what was presented.
 - **P.O.W.E.R. framework:** Since the 1st editions of our math with P.O.W.E.R. Learning textbooks, this research-based framework has informed the organization of chapters and sections to match the way that research shows students learn best.
 - **Rethink questions:** This is a *crucial* step in the learning process, and the “R” in P.O.W.E.R., yet it is the one that students overlook the most (or don’t even realize exists). Every exercise set is followed by a set of Rethink questions that require students to reflect upon what they have just done. These questions are also assignable in ConnectMath Hosted by ALEKS.
 - **Student success skills, emPOWERme, and Study Strategies:** Every chapter has a study skills theme with an emPOWERme survey and a Study Strategies page *in the book* so that instructors can address students’ weaknesses in this area. They are based upon Bob Feldman’s research, and can be done in any order. If instructors do not want to include them in their courses, they can be skipped easily.
- **Time-saving supplements, especially for adjunct instructors:** Many adjuncts teach at more than one school, and they don’t always have time to make materials specific to their classes and their students’ needs. We offer many author-created supplements with every textbook.
 - **Basic Skills Worksheets**—Help students improve their basic skills in class while using only 2 or 3 minutes of class time. Their confidence improves, too, when they see their basic skills improve. They start to believe that they *can* learn math!
 - **Section Worksheets**
 - **Worksheets to Tie Multiple Concepts Together**
 - **Guided Student Notes**
 - **NEW! Integrated Video & Study Guides (IVSGs)**—Great for flipped classrooms or any class, these require students to actively engage with the videos not only for doing examples but also for filling in information for elements such as definitions and procedures. Students *must* pay attention to complete these guides. The IVSGs are written for every *objective* in every section and designed so that most videos are 3 or 4 minutes long. The videos, and exercises from the video guides aligned to them, are assignable in ConnectMath Hosted by ALEKS.
 - **PowerPoints**
- **Enhanced exercise sets** with more conceptual questions
- **Videos for selected homework exercises**
- **Putting It All Together** sections in chapters where it is appropriate
- **Group Activities** in every chapter
- **Instructor Resource Manual**
- **ConnectMath Hosted by ALEKS and ALEKS**
- **ALEKS Prep for Introductory Algebra with P.O.W.E.R. Learning:** This preparatory tool focuses on prerequisite and introductory material for this text. The Prep products can be used during the first 6 weeks of a traditional course or in a corequisite course where students need to quickly narrow the gap in their skill and concept base. ALEKS Prep targets gaps in an individual student’s knowledge. It then provides assessment and learning directed toward individual student needs, an open response environment with realistic input tools, and unlimited online access on both PCs and MACs. A free trial is available at www.aleks.com/free_trial/instructor.

New to the 2nd Edition P.O.W.E.R. Series Books

- Approximately 1,000 new algorithmic exercises have been developed for the series within ConnectMath Hosted by ALEKS across the Prealgebra, Introductory Algebra, and Intermediate Algebra courses.
- Nearly 800 new online lecture videos for the series have been made for the eBooks in both ConnectMath Hosted by ALEKS and ALEKS, all of which are created and narrated by the authors themselves. These videos cover all Learning Objectives in the main print books and eBooks.
- A brand new workbook resource, the Integrated Video & Study Guide Workbook, has been created to be used in conjunction with the new online lecture videos, with a separate video guide provided for each new video. Practice exercises drawn from these video guides are assignable in ConnectMath Hosted by ALEKS.

Table of Contents

CHAPTER 1 The Real Number System and Geometry 1

Study Strategies: An Introduction to the P.O.W.E.R. Framework 2

Study Strategies: Using P.O.W.E.R. to Succeed in Your Course 4

Section 1.3 gives students the opportunity to relearn and practice geometry before getting to applications later in the book that will require this knowledge. It helps them make connections to other math skills they will need and breaks down the information into smaller pieces for easier understanding.

- Section 1.1** Review of Fractions 6
- Section 1.2** Exponents and Order of Operations 20
- Section 1.3** Geometry Review 26
- Section 1.4** Sets of Numbers and Absolute Value 38
- Section 1.5** Addition and Subtraction of Real Numbers 47
- Section 1.6** Multiplication and Division of Real Numbers 55
- Section 1.7** Properties of Real Numbers 63
- Section 1.8** Simplifying Algebraic Expressions 73
- Group Activity** 80
- emPOWERme: Measuring Your Math Confidence** 81
- Chapter Summary** 82
- Chapter 1 Review Exercises** 86
- Chapter 1 Test** 88

CHAPTER 2 Linear Equations and Inequalities in One Variable 90

Study Strategies: How to Read Math (and Other) Textbooks 91

Section 2.4 begins by teaching students the arithmetic before relating it to algebra. If students can't understand the arithmetic, they will find it next to impossible to solve an algebra problem.

- Section 2.1** Solving Linear Equations Part I 93
- Section 2.2** Solving Linear Equations Part II 104
- Section 2.3** Applications of Linear Equations 115
- Section 2.4** Applications Involving Percents 125
- Section 2.5** Geometry Applications and Solving Formulas 137
- Section 2.6** Applications of Linear Equations to Proportions, Money Problems, and $d = rt$ 152
- Section 2.7** Linear Inequalities in One Variable 165
- Group Activity** 177
- emPOWERme: Discover Your Reading Style** 178
- Chapter Summary** 179
- Chapter 2 Review Exercises** 183
- Chapter 2 Test** 185
- Cumulative Review for Chapters 1–2** 186

CHAPTER 3 Linear Equations in Two Variables 188

- Study Strategies: Time Management** 189
- Section 3.1** Introduction to Linear Equations in Two Variables 191
- Section 3.2** Graphing by Plotting Points and Finding Intercepts 205
- Section 3.3** The Slope of a Line 216
- Section 3.4** The Slope-Intercept Form of a Line 227
- Section 3.5** Writing an Equation of a Line 236
- Section 3.6** Introduction to Functions 248
- Group Activity** 262
- emPOWERme: Find Your Time Style** 264
- Chapter Summary** 265
- Chapter 3 Review Exercises** 270
- Chapter 3 Test** 273
- Cumulative Review for Chapters 1–3** 275

CHAPTER 4 Systems of Linear Equations and Inequalities in Two Variables 276

Study Strategies: Taking Notes in Class 277

- Section 4.1** Solving Systems by Graphing 279
- Section 4.2** Solving Systems by the Substitution Method 290
- Section 4.3** Solving Systems by the Elimination Method 298
Putting It All Together 307
- Section 4.4** Applications of Systems of Two Equations 309
- Section 4.5** Linear Inequalities in Two Variables 322
Group Activity 333
emPOWERme: Evaluate Your Class Notes 334
Chapter Summary 335
Chapter 4 Review Exercises 339
Chapter 4 Test 341
Cumulative Review for Chapters 1–4 342

CHAPTER 5 Rules of Exponents 343

Study Strategies: Taking Math Tests 344

Rules of Exponents is one of the more difficult topics for students. We give the material its own chapter and then break the topic out into more manageable pieces. Students are given a chance to *practice* the easier parts first before progressing to harder topics.

- Section 5.1** Basic Rules of Exponents
Part A: The Product Rule and Power Rules 346
Part B: Combining the Rules 353
- Section 5.2** Integer Exponents
Part A: Real-Number Bases 355
Part B: Variable Bases 358
- Section 5.3** The Quotient Rule 362
Putting It All Together 365
- Section 5.4** Scientific Notation 368
Group Activity 376
emPOWERme: Math Test Preparation Checklist 377
Chapter Summary 378
Chapter 5 Review Exercises 379
Chapter 5 Test 381
Cumulative Review for Chapters 1–5 381

CHAPTER 6 Polynomials 382

Study Strategies: Doing Math Homework 383

- Section 6.1** Addition and Subtraction of Polynomials 386
- Section 6.2** Multiplication of Polynomials 396
- Section 6.3** Special Products 402
- Section 6.4** Division of Polynomials 408
Group Activity 417
emPOWERme: What's Different About College? 418
Chapter Summary 420
Chapter 6 Review Exercises 421
Chapter 6 Test 423
Cumulative Review for Chapters 1–6 424

CHAPTER 7 Factoring Polynomials 426

Study Strategies: Making a Class Schedule 427

Section 7.2 helps students make the connection between arithmetic and factoring by starting off with exercises that are purely arithmetic.

- Section 7.1** The Greatest Common Factor and Factoring by Grouping 430
- Section 7.2** Factoring Trinomials of the Form $x^2 + bx + c$ 440
- Section 7.3** Factoring Trinomials of the Form $ax^2 + bx + c$ ($a \neq 1$) 447
- Section 7.4** Factoring Special Trinomials and Binomials 455
- Section 7.5** Factoring the Sum and Difference of Two Cubes 462
Putting It All Together 466
- Section 7.6** Solving Quadratic Equations by Factoring 469
- Section 7.7** Applications of Quadratic Equations 478
Group Activity 488
emPOWERme: Identify Major Attractions 491
Chapter Summary 493
Chapter 7 Review Exercises 495
Chapter 7 Test 497
Cumulative Review for Chapters 1–7 498

CHAPTER 8 Rational Expressions 499

Study Strategies: Know Your School 500

Finding the Least Common Denominator is a tough concept so we've broken it down and given it its own section, 8.3. If students have a chance to practice finding the least common denominator first, they will be more successful when they move on to Section 8.4

- Section 8.1 Simplifying Rational Expressions 504
- Section 8.2 Multiplying and Dividing Rational Expressions 516
- Section 8.3 Finding the Least Common Denominator 522
- Section 8.4 Adding and Subtracting Rational Expressions 530
- Putting It All Together 538
- Section 8.5 Simplifying Complex Fractions 543
- Section 8.6 Solving Rational Equations 552
- Section 8.7 Applications of Rational Equations and Variation 563
- Group Activity 578
- emPOWERme: My School 579
- Chapter Summary 580
- Chapter 8 Review Exercises 585
- Chapter 8 Test 588
- Cumulative Review for Chapters 1–8 589

CHAPTER 9 Roots and Radicals 590

Study Strategies: Working with a Study Group 591

We've broken down the material into bite-sized pieces before combining operations in Section 9.4.

- Section 9.1 Finding Roots 595
- Section 9.2 Simplifying Radicals: The Product and Quotient Rules 605
- Section 9.3 Adding and Subtracting Radicals 617
- Section 9.4 Combining Operations on Radicals 622
- Section 9.5 Dividing Radicals 628

- Section 9.6 Solving Radical Equations 640
- Section 9.7 Rational Exponents 650
- Group Activity 659
- emPOWERme: My Study Group Experience 660
- Chapter Summary 661
- Chapter 9 Review Exercises 664
- Chapter 9 Test 666
- Cumulative Review for Chapters 1–9 667

CHAPTER 10 Quadratic Equations 668

Study Strategies: Creating Visuals 669

- Section 10.1 Solving Quadratic Equations Using the Square Root Property 671
- Section 10.2 Solving Quadratic Equations by Completing the Square 678
- Section 10.3 Solving Quadratic Equations Using the Quadratic Formula 686
- Putting It All Together 694
- Section 10.4 Graphs of Quadratic Equations 697
- Section 10.5 Complex Numbers 705
- Group Activity 714
- emPOWERme: What's Your Learning Style? 715
- Chapter Summary 717
- Chapter 10 Review Exercises 720
- Chapter 10 Test 721
- Cumulative Review for Chapters 1–10 722

APPENDIX

- Section A.1 Decimals A-1
- Section A.2 Percents A-12
- Section A.3 Compound Inequalities in One Variable A-19
- Section A.4 Solving Systems of Three Equations and Applications A-27
- Section A.5 Compound Linear Inequalities in Two Variables Containing Or A-37
- Student Answer Appendix SA-1
- Instructor Answer Appendix (Annotated Instructor Edition only) IA-1
- Index I-1

Find these resources in your P.O.W.E.R. Tool Kit

In the textbook

- Work Hints*
- In-Class Examples*
- You Trys*
- 24-hr Problems
- Enhanced Exercise Sets
- Putting It All Together sections*
- Group Activities
- Study Strategies and emPOWERme in each chapter*

Supplements

- Basic Skills Worksheets*
- Section Worksheets*
- Worksheets to Tie Multiple Concepts Together*
- Guided Student Notes*
- Comprehensive video package, including Integrated Video & Study Guides and videos for selected exercises*
- Power Points
- Instructor Solution Manual
- Computerized Test Bank and files
- Instructor Resource Manual**
- ConnectMath Hosted by ALEKS and ALEKS*

*Descriptions of these resources are included in the following pages.

**Additional information about how to use the resources can be found in the Instructor Resource Manual.

WORK HINTS highlight important steps in working out a problem, point out places of common student errors, or give a study tip for learning. Pulling these tips out of the main text makes them more noticeable to students.

W Hint

Did you see that *like terms* are always lined up in the same columns?

$$\begin{array}{r}
 + 2 \overline{) 5x^2 + 13x + 6} \\
 \underline{-(5x^2 + 10x)} \\
 + 3x + 6 \\
 \underline{-(3x + 6)} \\
 0
 \end{array}$$

- 1) By what do we multiply x to get $3x$? 3
Write $+3$ above $+6$.
- 2) Multiply 3 by $(x + 2)$: $3(x + 2) = 3x + 6$
- 3) Subtract $(3x + 6) - (3x + 6) = 0$.
- 4) There are no more terms. The remainder is 0 .

The **IN-CLASS EXAMPLES** exactly mirror the examples in the book, giving instructors additional problems to use while teaching in class. They are available only in the Annotated Instructor Edition, and they align with the examples in the **Guided Student Notes**.

EXAMPLE 3

Factor $4n^3 - 12n^2 - 40n$ completely.

Solution

Ask yourself, “*Can I factor out a GCF?*” Yes. The GCF is $4n$.

$$4n^3 - 12n^2 - 40n = 4n(n^2 - 3n - 10)$$

YOU TRY problems follow almost every example in the book and exactly mirror the examples. After working through problems in class, the instructor can have the students do a **You Try** to practice on their own to reinforce the lesson. The **Examples**, **In-Class Examples**, and **You Try** problems are the same *types* of problems containing different numbers, providing consistency for students *and* instructors.

YOU TRY 2

For each rational expression, for what values of the variable

- i) does the expression equal zero?
- ii) is the expression undefined?

a) $\frac{v-6}{v+11}$

b) $\frac{9w}{w^2-12w+20}$

c) $\frac{x^2-25}{8}$

d) $\frac{1}{5q+4}$

The **PUTTING IT ALL TOGETHER** sections tie multiple concepts together and take students through the thought processes necessary for problem recognition. For example, after learning and practicing four methods for solving quadratic equations *individually*, the **Putting It All Together** section presents students with a group of quadratic equations in different forms and explains, step by step, how to decide which method to use to solve a particular equation.

Putting It All Together

P Prepare

What is your objective?

- 1 Decide Which Method to Use to Solve a Quadratic Equation

O Organize

How can you accomplish the objective?

- While following Example 1 on your own, make a chart that will help you identify when to use each of the four methods.
- Complete the given example on your own.
- Complete You Try 1.

W Work

Read the explanations, follow the example, take notes, and complete the You Try.

We have learned four methods for solving quadratic equations.

Methods for Solving Quadratic Equations

- 1) Factoring
- 2) Square root property
- 3) Completing the square
- 4) Quadratic formula

Video Package: Integrated Video and Study Guides as well as Exercise Videos

Would you like to flip your classroom or offer students a structured learning environment outside the classroom? The **Integrated Video and Study Guides**, IVSGs, allow instructors to do just that. New to this edition of the P.O.W.E.R. textbooks, the authors have created one **Integrated Video and Study Guide** for *each objective* in the book. Students are required to watch the video and fill out the **IVSG** as they go along, stopping along the way to do the similar *Your Turn* problems and *Think About It* conclusion questions designed for deeper conceptual thinking. Each companion video follows the procedures and examples, with detailed explanations aligning perfectly with the textbook. In addition to IVSG companion videos, hundreds of 3–5 minute **Exercise Video** clips show students how to solve various exercises from the textbook. All videos, as well as study strategies, are assignable in ConnectMath Hosted by ALEKS. Instructors also have the option to include the P.O.W.E.R. framework in assignments.

Play. Often, we have to take out a greatest common factor before applying other factoring techniques. Let's summarize how to factor by grouping and look at another example.

Procedure Steps for Factoring by Grouping

- 1) Before trying to factor by grouping, look at each term in the polynomial and ask yourself, "_____?" If so, factor out the GCF from all of the terms.
- 2) Make two groups of two terms so that each group has a common factor.
- 3) Take out the common factor in each group of terms.
- 4) Factor out the common binomial factor using the distributive property.
- 5) Check the answer by multiplying the factors.

Example 4: Factor $12t^3 + 12t^2 - 3t^2u - 3tu$ completely.

Pause and do Your Turn 4: Factor $8n^5 + 24n^4 - 16n^3 - 48n^2$ completely.

Think About It: Ms. Szymanski asks her students to factor $2x^3 + 10x^2 + 3x + 15$. Haeshin's answer is $(x + 5)(2x + 3)$, and Ilhan's answer is $(2x + 3)(x + 5)$. Who is right, and why?

Example 4: Factor $12t^3 + 12t^2 - 3t^2u - 3tu$ completely.

$$12t^3 + 12t^2 - 3t^2u - 3tu = 3t(4t^2 + 4t - tu - u) \quad \text{Factor out the GCF, } 3t$$

$$3t(4t^2 + 4t - tu - u)$$

Take out the common factor in each group.

$$3t[4t(t + 1) - u(t + 1)]$$

Factor out $t + 1$ using the distributive property.

$$= 3t(t + 1)(4t - u)$$

Check by multiplying: $3t(t + 1)(4t - u) = 3t(4t^2 + 4t - tu - u)$

$$= 12t^3 + 12t^2 - 3t^2u - 3tu$$

Your Turn 4: Factor $8n^5 + 24n^4 - 16n^3 - 48n^2$ completely.

common factor in each group. Factor out $t + 1$ using the distributive property. Check by multiplying. Now it's your turn. Pause the video

Student Success Skills in Every Chapter

Start the study skills discussion by having your students do the **emPOWERme** activity. Found in *every chapter* before the Chapter Summary, most **emPOWERme** activities take the form of a survey so that students can learn something about themselves with respect to the student success skill of the chapter. Follow this up with the **Study Strategies** found at the beginning of the chapter to help your students use the P.O.W.E.R. framework to acquire skills such as learning about their school, reading a math textbook, doing their homework effectively, or managing their time.

em me My School

Every school—whether it’s a high school, community college, college, or university—operates under its own set of rules and procedures. Understanding how your school works and where to go for help are essential parts of being successful in college. It’s important to understand how your school works so that, for example, you know where and when to turn in your financial aid application and you know where to get help if you have questions about choosing the classes you need for graduation. Take this survey to learn how well you know your school. Check all boxes that apply.

- I know the address of my school’s website.
- I can navigate the school’s website to find most information that I need.
- I am aware of whether my school has a handbook containing useful information.
- I have signed up to receive emergency campus messages by email, text, or automated phone call.
- I am aware of important dates such as when to register for classes, when tuition is due, and when financial aid forms are due.
- I know where to register for classes on campus.
- On campus, I know where to ask questions about financial aid.
- I can locate the bookstore.
- I know the location of the library.
- I know the difference between an adviser and a counselor.
- I know the location of the campus health center.
- I know the location of student services offices that might be of interest to me. Some examples are veterans’ support services, the office to help students with disabilities, and child care.
- I know the locations of all of my instructors’ offices as well as their office hours.
- I know the location of the tutoring center/math lab, and I know their procedures for getting help when I need it.
- I can locate the Testing Center and know its rules and hours of operation.
- I am aware of clubs, organizations, and activities on campus, and I know where to go to become involved in those that interest me.
- I know the location of the office where I can go if I have questions about or want help finding a job.

Think about the items that you have, and have *not*, checked in this survey. Which apply to you and might contribute to your success in college? In the Study Strategies at the beginning of this chapter, you will learn how to get to know your school.

Every Chapter Has a Student Success Theme

Find the **STUDY STRATEGIES** on the second page of every chapter. After doing the **emPOWERme** activity found before the Chapter Summary, continue the discussion of the chapter's student success skill by assigning or discussing the **Study Strategies**. Based on Bob Feldman's research, the **Study Strategies** explain, in easy-to-understand language, how to use each step of the P.O.W.E.R. framework to acquire student success skills such as getting to know your school, how to read a math textbook, and how to be a good time manager. The student success topic of each chapter is listed in the Table of Contents, and the topics can be done in any order.

POWER

Study Strategies

Know Your School

Understanding how your college or university works and where to find resources are critical components of being successful in college. Where do you go if you have a question about financial aid? Where can you find out about clubs that are of interest to you? Where do you go if you have questions about which classes you still need to take in order to graduate? Let's use the P.O.W.E.R. framework to help you get to know your school.

P Prepare

- I will learn about the different offices, resources, and services at my school as well as how my school works.

O Organize

- Complete the emPOWERme survey that appears before the Chapter Summary to learn how well you know your school. Think about what you know and what you still need to learn.
- Know your school's website. Identify important places on the website such as where you can register for classes, pay your tuition online, how to make an appointment to see an adviser, and where you can find information about the tutoring center.
- Identify places you need to know at your school: important offices where you can get questions answered, services offered at your school, organizations or clubs that you can join, etc.
- Identify important deadlines you should know such as registering for classes, paying tuition, and submitting financial aid applications.
- Identify academic services that you may want to use such as the library, a tutoring center, and a testing center.
- Identify support services that may be helpful to you such as the campus health center, a veterans' support office, an office to help students with disabilities, and a child care center.
- Be sure you have your syllabus for each class with information about the class as well as the location of your instructors' offices and their office hours.

W Work

- It's time to learn about some of the offices, resources, and support services at your school and to learn about some of the procedures. Some important, common offices and resources are listed here, but different schools have different names for these services. Some apply to all students, and some may not apply to you. For the items that apply to you, fill in the blanks with the information for *your* school.
- My school's website is* _____ . Among other things, know how you can register for classes and pay your tuition online.
- The office where I can go to register for classes is called the* _____ *office. Its location is* _____ . Know the deadlines for registering for classes.
- When you are choosing your classes, know where they are located. If the campus is large, be sure you have enough time to get from one class to the next.
- The office where I can go to ask questions about my tuition bill or to pay my bill in person is called the* _____ *office. Its location is* _____ . Know when tuition is due.
- The office where I can go to ask questions about financial aid, get financial aid forms, or turn in forms is called the* _____ *office. Its location is* _____ . Know when to get the financial aid process started and when the forms are due.
- The office where I can talk with someone about choosing a major or get help choosing classes is called the* _____ *office. Its location is* _____ .

E Evaluate

- Do you feel like you have a good understanding of how your school works and where to go for help? Did you learn the names and locations of the offices listed in the Work section?

R Rethink

- If you feel like you learned about the places and services that will be beneficial to you, be sure to use them when you need them! Also, guide classmates to the proper place when they need help.
- If you did not learn everything you think you need to know, ask one of your instructors or go to an information office at your school. Search the institution's website.
- Are there other services or resources that you feel like you need but did not learn about? Talk to one of your instructors and ask if there is such a place at your school.

_____ campuses have an office that helps veterans. *On my campus, that office is called* _____ . *Its location is* _____ .

_____ campuses have a child care center for students with children. *I can get* _____ *information about the child care center at* _____ .

_____ *Where I can go to find out about campus clubs, events, and organizations is called* _____ .

_____ institutions have a place where students can get help finding a job or preparing for an interview. *At my school, this office is called the* _____ *office. Its location is* _____ .



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500 CHAPTER 8 Rational Expressions

www.mhhe.com/messersmith

CHAPTER 8 Rational Expressions 501

Supplements Include a Suite of Ready-Made Worksheets

Want materials to use in class but don't have a lot of time to make them? Our package includes three types of author-created worksheets: **Basic Skills Worksheets**, **Section Worksheets**, and **Worksheets to Tie Multiple Concepts Together**. All are available in the student version (without answers) and instructor version (with answers), and *all worksheets are available as PDF or Word files so that instructors can download them and edit them as if they were their own Word documents*.

Worksheet 3C
MVF – Introductory Algebra

Name: _____

Find two numbers that...

MULTIPLY TO	and ADD TO	ANSWER
-27	-6	-9 and 3
72	18	
24	-11	
-4	3	
10	-7	
121	22	
-54	-3	
54	29	
16	-10	
30	17	
9	-6	
-8	-2	
21	10	
60	-19	
56	15	
-28	3	
-72	-6	
100	25	
-40	6	
11	-12	
20	12	
-35	-2	
77	18	
108	21	
-3	-2	

BASIC SKILLS WORKSHEETS

The **Basic Skills Worksheets** help students improve their basic skills while using only a few minutes, sometimes only 2 or 3, of class time. Their confidence improves, too, when they see their basic skills improve. Use these *before* reaching the topic where the basic skill is needed so that students can strengthen their weaknesses and be better prepared to learn new content. Each type of **Basic Skills Worksheet** comes in six different versions, incorporating more difficult concepts as they move from version A to version F.

These author-created, ready-made **Section Worksheets** and **Worksheets to Tie Multiple Concepts Together** can be used for extra practice in class, with students working individually or in a group, or can be used for students to take home. They come in student versions (without answers) and instructor versions (with answers).

SECTION WORKSHEETS

Every section of every book comes with at least one **Section Worksheet** to help instructors teach new content or to give students extra practice problems. The **Section Worksheets** *exactly match* the material in the section and are a great way to standardize instruction across a department.

WORKSHEETS THAT TIE MULTIPLE CONCEPTS TOGETHER

When appropriate, there are **Worksheets That Tie Multiple Concepts Together** to help students with problem recognition and differentiation. These worksheets allow students to practice multiple concepts together after having learned them individually.

Worksheet 3.3
MVF – Introductory Algebra

Name: _____

1) If a line contains the points (x_1, y_1) and (x_2, y_2) , write the formula for the slope of the line.

Find the slope of the line containing each pair of points.

- $(1, 3)$ and $(4, 9)$
- $(5, -2)$ and $(1, 6)$
- $(-3, -2)$ and $(2, 1)$
- $(-1, 4)$ and $(3, 4)$
- $(6, 3)$ and $(2, 4)$
- $(3, 7)$ and $(3, 0)$

Graph the line containing the given point and with the given slope.

- $(-4, -2)$; $m = \frac{3}{2}$
- $(0, 3)$; $m = -2$

Worksheet – Ch. 10 Putting It All Together
MVF – Introductory Algebra

Name: _____

You have now learned four ways to solve quadratic equations:

- Factoring
- Square Root Property
- Completing the Square
- Quadratic Formula

Keeping in mind these four methods, solve each of the following equations in the most efficient way.

- $k^2 - 6k - 3 = 0$
- $2y^2 = 2y + 5$
- $n^2 + n - 1 = 0$
- $a^2 = 3a + 54$
- $(z - 4)^2 = 9$
- $4m^2 - 2 = -5m$
- $(p + 8)^2 - 12 = 0$
- $3w^2 - 8 = 2w$
- $3x = 6x^2$
- $2y(3y + 4) + 6 = 4y + 7$
- $\frac{x}{2} + \frac{5}{x} = \frac{7}{2}$
- $\frac{t}{12} + \frac{3}{2t} = \frac{3}{4}$

Help Students Learn How to Take Good Notes

GUIDED STUDENT NOTES

Use the **Guided Student Notes** to help your students become better note-takers. Every section of the book has a ready-made corresponding **Guided Student Note** that mirrors the material in the section. These are “skeleton note outlines” that help students learn how to structure their notes. Because the structure is already given to students, they don’t have to write down *everything* that the instructors write in class so that students can concentrate better on what they are learning.

The **Guided Student Notes** save instructors time and help standardize the material that is taught across a math department. They come in both a P.O.W.E.R. format and a standard format and include answer keys. *The Guided Student Notes are available as PDF or Word files so that instructors can download them and edit them as if they were their own Word documents.*

Guided Student Notes
MVF - Introductory Algebra

Name _____

9.2 Simplifying Radicals: The Product and Quotient Rules

Prepare What are my goals for this section?

Organize What am I going to do to accomplish these goals?

Work

Definition: The Product Rule for Square Roots

Multiply. Assume that the variable represents a nonnegative real number.

1) $\sqrt{5} \cdot \sqrt{3}$ 2) $\sqrt{10} \cdot \sqrt{w}$

How to Simplify Expressions Containing Square Roots

Guided Student Notes
MVF - Introductory Algebra

Simplify completely.

3) $\sqrt{45}$ 5) $\sqrt{12}$

4) $\sqrt{500}$ 6) $\sqrt{15}$

Multiply and simplify.

7) $\sqrt{3} \cdot \sqrt{6}$ 8) $\sqrt{12} \cdot \sqrt{6}$

Definition: The Quotient Rule for Square Roots

Simplify completely.

9) $\sqrt{\frac{81}{64}}$ 10) $\sqrt{\frac{180}{5}}$



Looking for a consistent voice between text and digital? Problem solved!

McGraw-Hill Connect® Math Hosted by ALEKS® offers everything students and instructors need in one intuitive platform. ConnectMath is an online homework engine where the problems and solutions are consistent with the textbook authors' approach. It also offers integration with SmartBook, an assignable, adaptive eBook and study tool that directs students to the content they don't know and helps them study more efficiently. With ConnectMath, you get the tools you need to be the teacher you want to be.



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"I like that ConnectMath reaches students with different learning styles . . . our students are engaged, attend class, and ask excellent questions."
– Kelly Davis, South Texas College

Trusted Service and Support

A dedicated team of specialists and faculty consultants ensure that your ConnectMath implementation is seamless and painless . . . from start to finish.

ConnectMath Service and Support Offers:

- LMS integration that provides single sign-on capability and gradebook synchronization
- Industry-leading technical support and 99.97% uptime
- Resources for implementation, first day of class orientation, how-to videos and more
- Onsite seminars/workshops and webinars with McGraw-Hill and faculty consultants

How can ConnectMath help solve your students' challenges?

I like to learn by _____.

Whether it's reading, watching, discovering, or doing, ConnectMath has something for everyone. Instructors can create assignments that accommodate different learning styles, and students aren't stuck with boring multiple-choice problems. Instead they have a myriad of motivational learning and media resources at their fingertips. SmartBook delivers an interactive reading and learning experience that provides personalized guidance and just-in-time remediation. This helps students to focus on what they need, right when they need it.

I still don't get it. Can you do that problem again?

Because the content in ConnectMath is author-developed and goes through a rigorous quality control process, students hear one voice, one style, and don't get lost moving from text to digital. The high-quality, author-developed videos provide students ample opportunities to master concepts and practice skills that they need extra help with . . . all of which are integrated in the ConnectMath platform and the eBook.

How can ConnectMath help solve your classroom challenges?

I need meaningful data to measure student success!

From helping the student in the back row to tracking learning trends for your entire course, ConnectMath delivers the data you need to make an impactful, meaningful learning experience for students. With easy-to-interpret, downloadable reports, you can analyze learning rates for each assignment, monitor time on task, and learn where students' strengths and weaknesses are in each course area.

We're going with the _____ (flipped classroom, corequisite model, etc.) implementation.

ConnectMath can be used in any course setup. Each course in ConnectMath comes complete with its own set of text-specific assignments, author-developed videos and learning resources, and an integrated eBook that cater to the needs of your specific course. The easy-to-navigate home page keeps the learning curve at a minimum, but we still offer an abundance of tutorials and videos to help get you and your colleagues started.

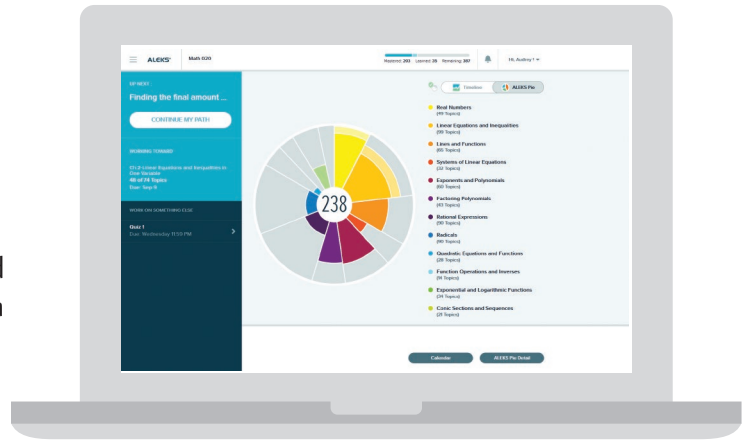


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Looking to motivate and engage students? Problem solved!

ALEKS® uses artificial intelligence to precisely map what each student knows, doesn't know, and is most ready to learn in a given course area. The system interacts with each student like a skilled human tutor, delivering a cycle of highly individualized learning and assessment that ensures mastery. Students are shown an optimal path to success, and instructors have the analytics they need to deliver a data-informed, impactful learning experience.



"ALEKS has helped to create the best classroom experience I've had in 36 years. I can reach each student with ALEKS."

– Tommy Thompson, Cedar Valley College, TX

How can ALEKS help solve your students' challenges?

I did all my homework, so why am I failing my exams?

The purpose of homework is to ensure mastery and prepare students for exams. ALEKS is the only adaptive learning system that ensures mastery through periodic assessments and delivers just-in-time remediation to efficiently prepare students. Because of how ALEKS presents lessons and practice, students learn by understanding the core principle of a concept rather than just memorizing a process.

I'm too far behind to catch up. - OR - I've already done this, I'm bored.

No two students are alike. So why start everyone on the same page? ALEKS diagnoses what each student knows and doesn't know, and prescribes an optimized learning path through your curriculum. Students only work on topics they are ready to learn, and they have a proven learning success rate of 93% or higher. As students watch their progress in the ALEKS Pie grow, their confidence grows with it.

How can ALEKS help solve your classroom challenges?

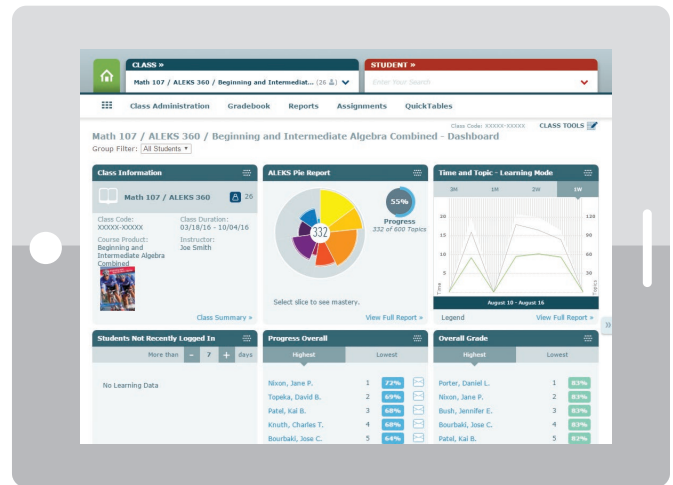
I need something that solves the problem of cost, time to completion, and student preparedness.

ALEKS is the perfect solution to these common problems. It provides an efficient path to mastery through its individualized cycle of learning and assessment. Students move through the course content more efficiently and are better prepared for subsequent courses. This saves both the institution and the student money. Increased student success means more students graduate.

My administration and department measure success differently. How can we compare notes?

ALEKS offers the most comprehensive and detailed data analytics on the market. From helping the student in the back row to monitoring pass rates across the department and institution, ALEKS delivers the data needed at all levels.

The customizable and intuitive reporting features allow you and your colleagues to easily gather, interpret, and share the data you need, when you need it.



The ALEKS Instructor Module offers a modern, intuitive interface to manage courses and track student progress.

Trusted Service and Support

A unique solution requires unique support. A dedicated team of specialists and faculty consultants ensure that your ALEKS implementation is seamless and painless . . . from start to finish.

ALEKS Service and Support Offers:

- LMS integration that provides single sign-on capability and gradebook synchronization
- Industry-leading technical support and 99.97% uptime
- Flexible courses that can align with any textbook and/or resources, for any classroom model
- Resources for implementation, first day of class orientation, how-to videos and more
- Onsite seminars/workshops and webinars with McGraw-Hill and faculty consultants

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The Real Number System and Geometry



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Math at Work: Landscape Architect

Jill Cavanaugh works as a landscape architect. She designs gardens, parks, and other outdoor spaces. To do her job, she uses multiplication, division, and geometry formulas on a daily basis.

For instance, when Jill is asked to create the landscape for a new house, her first task is to draw the plans. Often, the ground in front of the house will be dug out into shapes that include rectangles and circles, then shrubs and flowers will be planted, and mulch will cover the ground. To determine the volume of mulch that will be needed, Jill must use the formulas for the area of a rectangle and a circle and then multiply by the depth of the mulch. She will calculate the total cost of this landscaping job only after determining the costs of the plants, the mulch, and the labor.

In approaching a problem as complex as designing and creating the landscaping for an entire house, Jill takes a systematic, organized approach, working step by step to complete the job. This is the same technique you can use when you face a difficult math homework set or sit down for an exam. Keep your thinking and actions organized, and you will have a good chance to succeed.

In this chapter, we will review formulas from geometry as well as some concepts from arithmetic. We will also introduce P.O.W.E.R., a framework that you can use to help you meet challenges in this course and in virtually any other setting.

OUTLINE

Study Strategies: An Introduction to the P.O.W.E.R. Framework

Study Strategies: Using P.O.W.E.R. to Succeed in Your Course

- 1.1** Review of Fractions
- 1.2** Exponents and Order of Operations
- 1.3** Geometry Review
- 1.4** Sets of Numbers and Absolute Value
- 1.5** Addition and Subtraction of Real Numbers
- 1.6** Multiplication and Division of Real Numbers
- 1.7** Properties of Real Numbers
- 1.8** Simplifying Algebraic Expressions

Group Activity

emPOWERme: Measuring Your Math Confidence

Research shows that successful goal-achievers, and successful learners, do five things to achieve their goals: **P**repare, **O**rganize, **W**ork, **E**valuate, and **R**ethink. If we take the first five letters of those words, we get **P.O.W.E.R.**, as in the P.O.W.E.R. Learning Framework.

A great thing about this framework is that we can use it to do any task or achieve any goal whether it is cleaning out the basement, running a marathon, completing a project at work, or learning math! Let's learn what each step of P.O.W.E.R. means and apply it to an example to help you understand it.

P Prepare

- *Prepare means to explicitly state a goal.* Be very specific.
- Let's do an example: "I will train for and run a marathon one year from now."
- Keep in mind that there are long-term goals and short-term goals. Running in a marathon is the long-term goal, but we can set short-term goals along the way to help us achieve our ultimate goal of running in the race. We should train before running a marathon, so a short-term goal might be to increase the number of miles I am able to run every two weeks.

O Organize

- *Organize means to organize the physical and mental tools you need to achieve your goal.*
- A *mental tool* for running a marathon is planning: Where can I get information about training for a marathon? How will I fit the training into my schedule? Will I train at a gym as well as running on my own? Is there an app I can use to keep track of my progress? Do I need to change my diet so that I am more physically ready to run a race? Is there another person or a running group with whom I can train?
- I must gather or buy the *physical tools* needed such as new running shoes and clothes, a gym membership, a training schedule, healthy food that will give my body the nutrition it needs for training and running the race, a heart monitor, and a group of other people training for the marathon.

W Work

- *Work means to do the work that needs to be done to achieve the goal.*
- Time to train! I eat healthy foods and consult my training schedule. Some days I work out at the gym and run on the treadmill, and other days I run outside. I monitor my heart rate and keep track of the number of miles I run. Every Saturday, I meet with a group of people who are also training for the marathon so that we can work out together and share tips for preparing for and running the race.
- A year passes, and it is time for the race! I grab all of my gear, get to the starting line, and run the race.

E Evaluate

- *Evaluate means you should evaluate what you have done.* Did you achieve your goal or not?
- For our example, we look at our short-term and long-term goals. As I am training, I should think about whether I am achieving the short-term goal of increasing the number of miles I run every two weeks so that adjustments can be made, if necessary.
- The ultimate, and long-term goal, is running the marathon. Did I complete the race?

R Rethink

- *Rethink means to rethink and reflect upon your goal.* If you did *not* achieve your goal, ask yourself, "Why not?" If you did achieve your goal, ask yourself what you did *right*.
- First, reflect upon the *short-term goal* along the way. If I *did* increase the number miles I ran every two weeks during training, I should continue to train as I have but also think about ways that I can improve. If I did *not* increase the number of miles

I could run, I should ask myself, “*Why not?*” Did I fail to stick to my training schedule? Was I cheating on my training diet? Were my mileage goals unrealistic, and do they need to be adjusted? I should continually tweak the training process so that it is the best it can be.

- Now, reflect upon the ultimate goal of running in the marathon. If I completed the race, I should think about how I trained so that I can use some of the same strategies in the future. If I did *not* complete the race, I should ask myself why: Did I have an appropriate training schedule, and did I follow it? Did I have the right equipment to prepare for and run the race? If I plan to run this race again, I should think about my preparation and make adjustments.
- Maybe some factors were out of my control: Was it unusually hot on race day? Did that affect how far I could run? If so, maybe completing 18 miles out of the 26.2 miles was, actually, a successful race. Sometimes, we need to rethink the goal.

You can apply these same steps to help you be a successful student! Throughout the book we will use the P.O.W.E.R. Learning Framework to learn math *and* to acquire study skills to help us succeed in *any* course. And, of course, you can use P.O.W.E.R. outside of school to help you achieve *any* goal!



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Now we know that **P.O.W.E.R.** stands for **Prepare, Organize, Work, Evaluate, and Rethink.** We have learned how to apply the steps to a real-life situation, *training for and running a marathon*, but how do we apply the steps to learning math? **Let's apply the P.O.W.E.R. framework to help a fictional student, Derek, succeed in this course.**

P Prepare

- **Prepare** means to *explicitly state a goal.* Remember, be specific.
- Derek's goal is **"I will make at least a B in this course."**

O Organize

- **Organize** means to *organize the physical and mental tools you need to achieve your goal.*
- Some *physical tools* Derek will need are a backpack, a book (a print book or an e-book), a notebook, regular pencils, colored pencils (for taking good notes), and a quiet place to study.
- Some *mental tools* he will need are a positive attitude, a commitment to go to class every day and do his homework, good basic skills such as knowing the multiplication facts from 1 to 12, good time management so that he has enough time to work, go to class, and do his homework, and a knowledge of the locations of his instructor's office and the tutoring center.

W Work

- **Work** means to *do the work that needs to be done to achieve the goal.*
- Derek now has to **do the work** so that he can make at least a B in the course! That work includes going to class every day, putting away his cell phone, reviewing multiplication facts that he has trouble remembering, taking good notes, asking questions in class, studying in a place without distractions, rereading his notes when he gets home, reading the book and *writing out the examples as he reads them*, finishing his homework on time, and going to the tutoring center or to his instructor's office when he needs extra help.

E Evaluate

- **Evaluate** means you *evaluate what you have done.* Did you achieve your goal or not?
- Derek should **evaluate** how he is doing *as the term goes along* so that he can make adjustments, if necessary. He should ask himself, *"Am I making the kinds of grades that I need to make on the quizzes and tests?"*
- He should **evaluate** how he has done at the *end of the term.* He should ask himself, *"Did I make at least a B in the course?"*

R Rethink

- **Rethink** means to *rethink and reflect upon your goal.* If you did *not* achieve your goal, ask yourself, *"Why not?"* If you did achieve your goal, ask yourself what you did *right.*
- First, Derek should **rethink** *throughout the term.* If he is doing well on his quizzes and tests, that's great! He should ask himself, *"What have I been doing right so far?"* so that he continues to do those things. If he is not making a B at this point, he should ask himself, *"Why not?"* as well as questions such as *"Did I miss too many classes? Did I put off doing my homework until the last minute? Should I go in for help more often? What can I do differently to improve before the end of the term?"* Then, he can make adjustments so that he still has time to learn the material and improve his grade.
- Of course, Derek should **rethink** *at the end of the term.* If he made an A or a B in the course, then he has achieved his goal! *Still,* he should stop, think, and ask himself what he did to be successful so that he can apply similar strategies in the future. If he did *not* make at least a B in the course, he should ask himself, *"Why not?"* as well as ask himself the questions in the previous bullet. This way, he can do things differently in the future.

This is just one example of how you can apply the **P.O.W.E.R.** framework to learning math (or anything)! In the following chapters we will learn more *specific* skills to help you learn math and acquire study skills that will help you in any of your courses and in life.

Chapter 1 **POWER** Plan

P Prepare

O Organize

What are your goals for Chapter 1?	How can you accomplish each goal?
1 Be prepared before and during class.	<ul style="list-style-type: none"> • Don't stay out late the night before class, and be sure to set your alarm! • Bring a pencil, notebook paper, and textbook to class. • Avoid distractions by turning off your cell phone during class. • Pay attention, take good notes, and ask questions. • Complete your homework on time and ask questions on problems you do not understand.
2 Understand the homework to the point where you could do it without needing any help or hints.	<ul style="list-style-type: none"> • Read the directions and show all of your steps. • Go to the instructor's office for help. • Rework homework and quiz problems, and find similar problems for practice.
3 Learn and understand what each letter in P.O.W.E.R. represents so that you can apply it to learning math, acquiring study skills, and use in your everyday life.	<ul style="list-style-type: none"> • Complete the emPOWERme that appears before the Chapter Summary. • Read the Study Strategy that explains how to use the P.O.W.E.R. framework. • Explain what each letter in P.O.W.E.R. stands for <i>and</i> what each step means.
4 Write your own goal. _____ _____	<ul style="list-style-type: none"> • _____ _____
What are your objectives for Chapter 1?	How can you accomplish each objective?
1 Review the operations with fractions.	<ul style="list-style-type: none"> • Review the rules for writing a fraction in lowest terms. Understand how to multiply, divide, add, and subtract fractions and mixed numbers.
2 Learn to use exponents.	<ul style="list-style-type: none"> • Exponents indicate repeated multiplication of the base. • Be able to quickly evaluate and simplify commonly used powers.
3 Learn to use the order of operations in all situations.	<ul style="list-style-type: none"> • Create your own acronym like Please Excuse My Dear Aunt Sally to help you know how to evaluate mathematical expressions.
4 Review the basic concepts of geometry.	<ul style="list-style-type: none"> • Learn all classifications of angles, lines, and triangles. • Learn all formulas for finding perimeter, circumference, area, and volume of common shapes.
5 Understand sets of numbers and absolute value.	<ul style="list-style-type: none"> • Be able to identify as well as compare and contrast natural numbers, whole numbers, integers, rational numbers, and irrational numbers. • Be able to compare real numbers using a number line as well as symbols. • Find the absolute value of a number, and know what it means.
6 Be able to add, subtract, multiply, and divide real numbers, and translate English expressions to mathematical expressions.	<ul style="list-style-type: none"> • Know the rules for adding numbers of the same sign. • Know how to use absolute value when subtracting numbers or adding numbers with different signs. • Know the procedures for multiplying and dividing real numbers and when the results will be positive or negative.
7 Know the properties of real numbers, and apply them to algebraic expressions.	<ul style="list-style-type: none"> • Be able to use the commutative, associative, identity, inverse, and distributive properties. • Use the properties to help you combine like terms in an expression.
8 Write your own goal. _____ _____	<ul style="list-style-type: none"> • _____ _____

W Work

Read Sections 1.1–1.8,
and complete the exercises.

E Evaluate

Complete the Chapter Review
and Chapter Test. How did
you do?

R Rethink

- How did you perform on the goals for the chapter? If you had the chance to do this chapter over, what would you do differently?
- Think of a job you might like to have and describe how you would need to use what you have just learned to effectively do that job.
- How has the P.O.W.E.R. framework helped you master the objectives of this chapter? Where else could you use this framework? Make it a point to use P.O.W.E.R. to complete another task this week.

1.1 Review of Fractions

P Prepare**O Organize**

What are your objectives for Section 1.1?	How can you accomplish each objective?
1 Understand What a Fraction Represents	<ul style="list-style-type: none"> • Write your own definition of a fraction, and include the words <i>numerator</i> and <i>denominator</i>. • Complete Example 1 on your own. • Complete You Try 1.
2 Write Fractions in Lowest Terms	<ul style="list-style-type: none"> • Know how to write the prime factorization of a number. • In your own words, write a procedure for writing a fraction in lowest terms. • Complete the examples on your own. • Complete You Trys 2–6.
3 Multiply and Divide Fractions	<ul style="list-style-type: none"> • In your own words, write a procedure for multiplying fractions and mixed numbers. • Be able to find the reciprocal of a number. • In your own words, write a procedure for dividing fractions and mixed numbers. • Complete the examples on your own. • Complete You Trys 7 and 8.
4 Add and Subtract Fractions	<ul style="list-style-type: none"> • Understand that to add or subtract fractions, the fractions must have a common denominator. • Know how to find a least common denominator for a group of fractions. • Be able to rewrite a fraction with a different denominator. • In your own words, write a procedure for Adding and Subtracting Fractions with Unlike Denominators. • Complete the examples on your own. • Complete You Trys 9–13.

Read the explanations, follow the examples, take notes, and complete the You Trys.

Why review fractions and arithmetic skills? Because the manipulations done in arithmetic and with fractions are precisely the same skills needed to learn algebra.

Let's begin by defining some numbers used in arithmetic:

Natural numbers: 1, 2, 3, 4, 5, ...

Whole numbers: 0, 1, 2, 3, 4, 5, ...

Natural numbers are often thought of as the counting numbers. **Whole numbers** consist of the natural numbers and zero.

Natural and whole numbers are used to represent complete quantities. To represent a part of a quantity, we can use a fraction.

1 Understand What a Fraction Represents

What is a fraction?

Definition

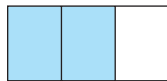
A **fraction** is a number in the form $\frac{a}{b}$, where $b \neq 0$. a is called the **numerator**, and b is the **denominator**.

Note

- 1) A fraction describes a part of a whole quantity.
- 2) $\frac{a}{b}$ means $a \div b$.

EXAMPLE 1

What part of the figure is shaded?



Solution

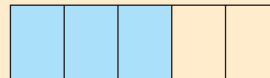
The whole figure is divided into three equal parts. Two of the parts are shaded. Therefore, the part of the figure that is shaded is $\frac{2}{3}$.

$\frac{2}{3}$ → Number of shaded parts

3 → Total number of equal parts in the figure

YOU TRY 1

What part of the figure is shaded?



2 Write Fractions in Lowest Terms

A fraction is in **lowest terms** when the numerator and denominator have no common factors except 1. Before discussing how to write a fraction in lowest terms, we need to know about factors.

Consider the number 12.

$$12 = 3 \cdot 4$$

↑ ↑ ↑
Product Factor Factor

3 and 4 are **factors** of 12. (When we use the term *factors*, we mean natural numbers.) Multiplying 3 and 4 results in 12. 12 is the **product**.

Does 12 have any other factors?

EXAMPLE 2

Find all factors of 12.

Solution

$$\begin{array}{ll} 12 = 3 \cdot 4 & \text{Factors are 3 and 4.} \\ 12 = 2 \cdot 6 & \text{Factors are 2 and 6.} \\ 12 = 1 \cdot 12 & \text{Factors are 1 and 12.} \end{array}$$

These are all of the ways to write 12 as the product of two factors. The factors of 12 are 1, 2, 3, 4, 6, and 12.

YOU TRY 2

Find all factors of 30.

We can also write 12 as a product of *prime numbers*.

Definition

A **prime number** is a natural number whose only two *different* factors are 1 and itself. (The factors are natural numbers.)

EXAMPLE 3

Is 7 a prime number?

Solution

Yes. The only way to write 7 as a product of natural numbers is $1 \cdot 7$.

YOU TRY 3

Is 19 a prime number?

Definition

A **composite number** is a natural number with factors other than 1 and itself. Therefore, if a natural number is not prime, it is composite (with the exception of 0 and 1).


Note

The numbers 0 and 1 are neither prime nor composite.

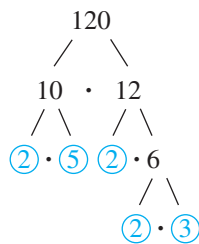
[YOU TRY 4]

- a) What are the first six prime numbers?
b) What are the first six composite numbers?

To perform various operations in arithmetic and algebra, it is helpful to write a number as the product of its **prime factors**. This is called finding the **prime factorization** of a number. We can use a **factor tree** to help us find the prime factorization of a number.

EXAMPLE 4

Write 120 as the product of its prime factors.

Solution

Think of *any* two natural numbers that multiply to 120.

10 and 12 are not prime, so write them as the product of two factors.

Circle the primes.

6 is not prime, so write it as the product of two factors. The factors are primes. Circle them.

W Hint

Write a procedure for using a factor tree to write the prime factorization of a number.

Prime factorization: $120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$.

[YOU TRY 5]

Use a factor tree to write each number as the product of its prime factors.

- a) 30 b) 175

Let's return to writing a fraction in lowest terms.

EXAMPLE 5

Write $\frac{48}{42}$ in lowest terms.

Solution

$\frac{48}{42}$ is an **improper fraction**. A fraction is *improper* if its numerator is greater than or equal to its denominator. We will use two methods to express this fraction in lowest terms.

Method 1

Using a factor tree to get the prime factorizations of 48 and 42 and then dividing out common factors, we have

$$\frac{48}{42} = \frac{\overset{1}{2} \cdot 2 \cdot 2 \cdot 2 \cdot \overset{1}{2}}{\underset{1}{2} \cdot \underset{1}{2} \cdot 7} = \frac{2 \cdot 2 \cdot 2}{7} = \frac{8}{7} \text{ or } 1\frac{1}{7}$$

The answer may be expressed as an improper fraction, $\frac{8}{7}$, or as a **mixed number**, $1\frac{1}{7}$, as long as each is in lowest terms.

W Hint

Describe two ways to write a fraction in lowest terms.